Stochastic Gradient Push for Distributed Deep Learning

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joint work with Mido Assran, Mike Rabbat, Nicolas Ballas (Facebook AI Research)

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Overview

1. Introduction
   - Problem formulation
   - Motivation for Decentralized Topologies
   - Related Work: From Average Consensus to Optimization
2. The Algorithm: Stochastic Gradient Push
   - PushSum Protocol for consensus and SGD
   - Theoretical guarantees: Non-convex functions
3. Numerical Experiments
   - Training ResNet50 on ImageNet Classification task
   - Comparison with state-of-the-art
4. Conclusion & Future Directions of Research

Problem Formulation

Network of n nodes (machines/agents) cooperates to solve the stochastic consensus optimization problem:

$$\min_{x\in\mathbb{R}^d} f(x) = \frac{1}{n}\sum_{i=1}^{n} \underbrace{\mathbb{E}_{\xi\sim D_i} F_i(x;\xi)}_{f_i(x)}$$

Distributed Setting

- f is described by **too much data to be stored on a singe computer**
- a **single computer is not powerful enough** for the task at hand and we have access to multiple computers.

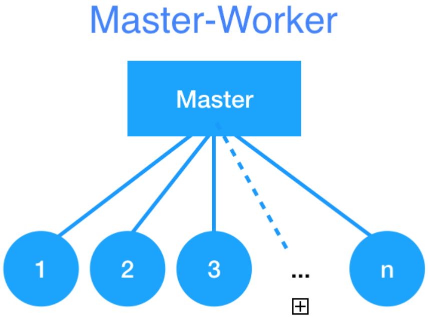
- Node $i \in [n]$ has local data following a distribution D_i.
- Nodes can locally evaluate stochastic gradients $\nabla F_i(\mathbf{x};\xi_i)$, $\xi_i \sim D_i$
- Communication is required to access information from other nodes.

GOAL: Each node evaluates the vector x that minimize $f(x)$.

On Parallel Stochastic Gradient Descent

Workhorse Algorithm: Stochastic Gradient Descent

$$x^{k+1} = x^k - \gamma^k \left(\frac{1}{n} \sum_i^n \nabla F_i(\mathbf{x}; \xi_i) \right)$$



- Leverage parallel computing resources to speed up training.
- Central node aggregates the gradient computed from the other nodes.

Communication Bottleneck

Communication traffic jam on the central node.

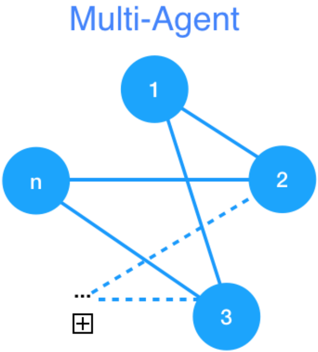


Figure:
$$x_i^{k+1} = x_i^k - \gamma^k \left(\frac{1}{N_i^k} \sum_{j \in N_i^k} \nabla F_j(x_j^k; \xi_i) \right)$$

On decentralized methods

- First developed by control theory/signal processing communities.
- Topology of network is known (application oriented).
- Goal: Design algorithm that converges for this topology.

Difference in Deep learning:

For training DNNs select the network the benefits the training!

Goal: Faster convergence!

Validation accuracy!

On Decentralized Methods

The decentralized optimization methods for machine learning and deep learning tasks based on average consensus methods.

Average Consensus (AC) Problem

Let each node $i \in [n]$ to "know" a private value $c_i \in \mathbb{R}$.
Goal: All nodes compute $\bar{c} := \frac{1}{n}\sum_i c_i$, in a distributed fashion.

Decentralized Protocols:

1. D-PSGD (PushPull parameter aggregation, neighboring nodes)
   AC: [Xiao et al 2005] $\rightarrow$ SC: [Nedic, Ozdaglar 2009]
   $\rightarrow$ DNNs: Lian et al. Neurips 2017. (symmetric communication)

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   AC: [Boyd et al 2006] [Loizou et al. 2019] $\rightarrow$ SC: [Nedic et al. 2010]
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   $\rightarrow$ DNNs: Lian et al. ICML 2018. (symmetric communication)
3. Push-Sum (directed, time varying graphs)
   AC: [Kempe et al. 2003] $\rightarrow$ SC: [Nedic, Olshevsky 2016]
   $\rightarrow$ DNNs: This Work

Push-Sum Protocol [Kempe et al. FOCS 2003]

Let $x_i^0 \in \mathbb{R}^d$ be a vector at node i. Goal:Evaluate $\frac{1}{n}\sum_i^n x_i^0$

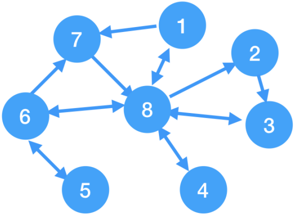


Figure: Directed graph.

Column stochastic matrix $\boldsymbol{P}$:

$$\boldsymbol{P}_{i,j} > 0 \Leftrightarrow (j \to i) \in \mathcal{E}$$
$$\sum_j \boldsymbol{P}_{i,j} = 1$$

Push-Sum Protocol:

1. Initialize $x_i^0 \in \mathbb{R}^d$ and $\omega_i^0 = 1$.
2. Let $\mathbf{X}^0 \in R^{n\times d}$ and $\omega \in R^n$
3. Iterate for $k \geq 0$:
   - $\mathbf{X}^k = \boldsymbol{P}\mathbf{X}^{k-1} = \boldsymbol{P}^k\mathbf{X}^0$
   - $\omega^k = \boldsymbol{P}\omega^{k-1} = \boldsymbol{P}^k\omega^0$
   - $z_i^k = \frac{x_i^k}{\omega_i^k} \to (\frac{1}{n}\sum_i^n x_i^0)$

Thus the update at node i:

$$\boldsymbol{x}_i^{k+1} = \sum_{j=1}^n p_{i,j}^k \boldsymbol{x}_j^k = \sum_{j\in\mathcal{N}_i^{\text{in}(k)}} p_{i,j}^k \boldsymbol{x}_j^k \ .$$

Stochastic Gradient Push (SGP)

Initialize:
$\gamma > 0$, $\boldsymbol{x}_i^{(0)} = \boldsymbol{z}_i^{(0)} \in \mathbb{R}^d$ and $w_i^{(0)} = 1$ for all nodes $i \in \{1, 2, \ldots, n\}$
For iterations $k = 0, 1, 2, \cdots, K$, at node i do:

1. **Local Update:** (SGD/momentum/Adam)
   - Sample new mini-batch $\xi_i^{(k)} \sim \mathcal{D}_i$, compute $\nabla \boldsymbol{F}_i(\boldsymbol{z}_i^{(k)}; \xi_i^{(k)})$
   - Update mini-batch gradient : $\boldsymbol{x}_i^{(k+\frac{1}{2})} = \boldsymbol{x}_i^{(k)} - \gamma \nabla \boldsymbol{F}_i(\boldsymbol{z}_i^{(k)}; \xi_i^{(k)})$
2. **Communication:** (τ-overlap SGP)
   - Send $\left(p_{j,i}^{(k)} \boldsymbol{x}_i^{(k+\frac{1}{2})}, p_{j,i}^{(k)} w_i^{(k)}\right)$ to out-neighbors $j \in \mathcal{N}_i^{\text{out}(k)}$
   - Receive $\left(p_{i,j}^{(k)} \boldsymbol{x}_j^{(k+\frac{1}{2})}, p_{i,j}^{(k)} w_j^{(k)}\right)$ from in-neighbors $j \in \mathcal{N}_i^{\text{in}(k)}$
3. **Aggregate:** (Push-Sum Protocol)
   - $\boldsymbol{x}_i^{(k+1)} = \sum_{j \in \mathcal{N}_i^{\text{in}(k)}} p_{i,j}^{(k)} \boldsymbol{x}_j^{(k+\frac{1}{2})}$
   - $w_i^{(k+1)} = \sum_{j \in \mathcal{N}_i^{\text{in}(k)}} p_{i,j}^{(k)} w_j^{(k)}$
   - $\boldsymbol{z}_i^{(k+1)} = \boldsymbol{x}_i^{(k+1)} / w_i^{(k+1)}$

Algorithm 1 Stochastic Gradient Push (SGP), [Nedić, Olshevsky 2016]

Require: Initialize $\gamma > 0$, $\boldsymbol{x}_i^{(0)} = \boldsymbol{z}_i^{(0)} \in \mathbb{R}^d$ and $w_i^{(0)} = 1$ for all nodes $i \in \{1, 2, \dots, n\}$

for $k = 0, 1, 2, \cdots, K$ **do**

  Sample new mini-batch $\xi_i^{(k)} \sim \mathcal{D}_i$ from local distribution

  Compute a local stochastic mini-batch gradient at $\boldsymbol{z}_i^{(k)}$: $\nabla \boldsymbol{F}_i(\boldsymbol{z}_i^{(k)}; \xi_i^{(k)})$

  $\boldsymbol{x}_i^{(k+\frac{1}{2})} = \boldsymbol{x}_i^{(k)} - \gamma \nabla \boldsymbol{F}_i(\boldsymbol{z}_i^{(k)}; \xi_i^{(k)})$

  Send $\left(p_{j,i}^{(k)} \boldsymbol{x}_i^{(k+\frac{1}{2})}, p_{j,i}^{(k)} w_i^{(k)}\right)$ to out-neighbors $j \in \mathcal{N}_i^{\text{out}(k)}$;
   receive $\left(p_{i,j}^{(k)} \boldsymbol{x}_j^{(k+\frac{1}{2})}, p_{i,j}^{(k)} w_j^{(k)}\right)$ from in-neighbors $j \in \mathcal{N}_i^{\text{in}(k)}$

  $\boldsymbol{x}_i^{(k+1)} = \sum_{j \in \mathcal{N}_i^{\text{in}(k)}} p_{i,j}^{(k)} \boldsymbol{x}_j^{(k+\frac{1}{2})}$

  $w_i^{(k+1)} = \sum_{j \in \mathcal{N}_i^{\text{in}(k)}} p_{i,j}^{(k)} w_j^{(k)}$

  $\boldsymbol{z}_i^{(k+1)} = \boldsymbol{x}_i^{(k+1)} / w_i^{(k+1)}$

end for

The Problem: Main Assumptions

$$\min_{x \in \mathbb{R}^d} f(x) = \frac{1}{n} \sum_{i=1}^{n} \underbrace{\mathbb{E}_{\xi \sim D_i} F_i(x; \xi)}_{f_i(x)}$$

Main Assumptions

- *L-smooth:* $\exists\ L > 0$: $\|\nabla f_i(\boldsymbol{x}) - \nabla f_i(\boldsymbol{y})\| \leq L\|\boldsymbol{x} - \boldsymbol{y}\|$.
- *Bounded variance:* $\mathbb{E}_{\xi \sim D_i} \|\nabla F_i(\boldsymbol{x}; \xi) - \nabla f_i(\boldsymbol{x})\|^2 \leq \sigma^2 \quad \forall i, \forall \boldsymbol{x}$
- *Similar objectives:* $\frac{1}{n} \sum_{i=1}^{n} \|\nabla f_i(\boldsymbol{x}) - \nabla f(\boldsymbol{x})\|^2 \leq \zeta^2 \quad \forall \boldsymbol{x}$.
- *Mixing Connectivity:* $\exists\ B, \Delta > 0$: graph with edge set $\bigcup_{k=lB}^{(l+1)B-1} E^{(k)}$ is strongly connected and has diameter at most Δ for every $l \geq 0$. Here $E^{(k)} = \{(i,j) \colon p_{i,j}^{(k)} > 0\}$.
- *Bounded Delays (Overlap-SGP)*: $\exists\ \tau \in \mathbb{Z}_+$, such that the delay, satisfies $k' - k \leq \tau$

Relation of SGP to other approaches

Parallel SGD (AllReduce gradient aggregation / all nodes)

Topology: fully-connected at every iteration

Uniform mixing weights: That is, $p_{j,i}^{(k)} = 1/n$ for all $i, j = 1, \ldots, n$

In this case the push-sum weight $w_i^k = 1, \quad \forall k, \forall i \in [n]$

D-PSGD

Topology: Static, undirected, and connected at every iteration

Symmetric mixing weights: $p_{j,i}^{(k)} = p_{i,j}^{(k)}$ for all (i,j) : $\sum_j p_{j,i}^{(k)} = 1$;

In this case the push-sum weight $w_i^k = 1, \quad \forall k, \forall i \in [n]$

Directed time-varying graphs

Topology: Directed, potentially time-varying, and B-strongly connected

Nodes choose *mixing weights* $p_{j,i}^{(k)}$ independently of one another;

In this case, push-sum weight w_i^k may differ between nodes at any given iteration, and we obtain a new operating regime.

Main Theoretical Contributions

Theorem (Convergence of $\overline{\boldsymbol{x}}^{(k)}$)

Suppose that main assumptions hold. Run SGP for K iterations with step-size $\gamma = \sqrt{n/K}$. Let $f^* = \inf_{\boldsymbol{x}} f(\boldsymbol{x})$ and assume that $f^* > -\infty$. There exist constants C and $q \in [0, 1)$, which depend on Δ, $\boldsymbol{P}^{(k)}$ and τ such that when:

$$K \geq \max \left\{ \frac{nL^4 C^4 60^2}{(1-q)^4}, \frac{L^4 C^4 P_1^2 n}{(1-q)^4 (f(\overline{\boldsymbol{x}}^{(0)}) - f^* + \frac{L\sigma^2}{2})^2}, \frac{L^2 C^2 n P_2}{(1-q)^2 (f(\overline{\boldsymbol{x}}^{(0)}) - f^* + \frac{L\sigma^2}{2})}, n \right\}$$

then

$$\frac{\sum_{k=0}^{K-1} \mathbb{E} \left\| \nabla f(\overline{\boldsymbol{x}}^{(k)}) \right\|^2}{K} \leq \frac{12(f(\overline{\boldsymbol{x}}^{(0)}) - f^* + \frac{L\sigma^2}{2})}{\sqrt{nK}}.$$

where $\overline{\boldsymbol{x}}^{(k)} = \frac{1}{n} \sum_{i=1}^{n} \boldsymbol{x}_i^{(k)}$ (average of the nodes' parameters).

Remark: Centralized parallel SGD converges also with $O(1/\sqrt{nK})$.

Main Theoretical Contributions

Theorem (Convergence to stationary point)

Under the same assumptions,

$$\frac{1}{nK}\sum_{k=0}^{K-1}\sum_{i=1}^{n}\mathbb{E}\left\|\overline{\boldsymbol{x}}^{(k)}-\boldsymbol{z}_i^{(k)}\right\|^2 \leq O\left(\frac{1}{K}+\frac{1}{K^{3/2}}\right),$$

and

$$\frac{1}{nK}\sum_{k=0}^{K-1}\sum_{i=1}^{n}\mathbb{E}\left\|\nabla f(\boldsymbol{z}_i^k)\right\|^2 \leq O\left(\frac{1}{\sqrt{nK}}+\frac{1}{K}+\frac{1}{K^{3/2}}\right).$$

As K grows:

- Variables $\boldsymbol{z}_i^{(k)} \longrightarrow \overline{\boldsymbol{x}}^{(k)}$,
- Convergence to a stationary point.
- For fixed n and large K, the $1/\sqrt{nK}$ term will dominate the other factors.

Numerical Evaluation

- Training ResNet50 on ImageNet Classification task
- System: 8 GPUs/nodes. Look ar scaling from 4-32 nodes (32-256 GPUs)
- Communication over 10 Gbps Ethernet (high latency scenario) and 100 Gbps Infiniband networks (no communication bottleneck)
- Comparison with State-of-the-art:
  - (i) AllReduced-based SGD [Goyal et al. 2017]
  - (ii) D-PSGD, decentralized push-pull stochastic gradient descent [Lian et al. NIPS 2017]
  - (iii) AD-PSGD , asynchronous decentralized push-pull stochastic gradient descent [Lian et al. ICML 2018]
- **Code:**
  `https://github.com/facebookresearch/stochastic_gradient_push`

Graph Topology: Directed Exponential Graph

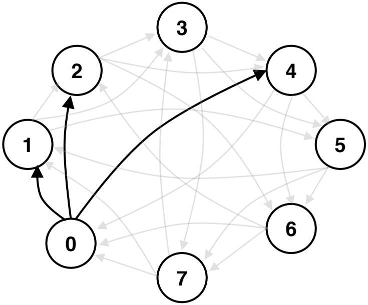
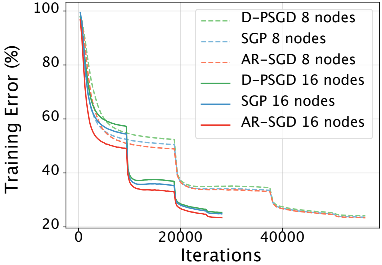
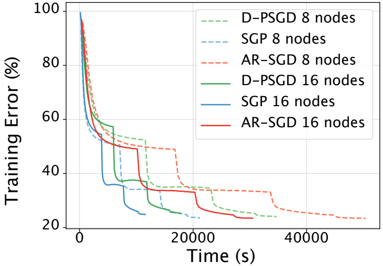


Figure: 8-node directed exponential graph, highlighting node 0's out-neighbours.

- **Cyclic strategy:**
  Each node sends and receives one message per update.
  Node i sends to:
  - $i + 2^0$mod n
  - $i + 2^1$mod n
  - $\ldots$
  - $i + 2^{\log_2(n-1)}$mod n
- Mixing Matrices $\mathbf{P}^k$
  - Node i choose its mixing weights (column i of $\mathbf{P}^k$)
  - Uniform mixing weights.
  - For one-peer-per-node case: Each column of $\mathbf{P}^k$ has exactly two non-zero entries, both equal to $1/2$.

Scaling and Convergence





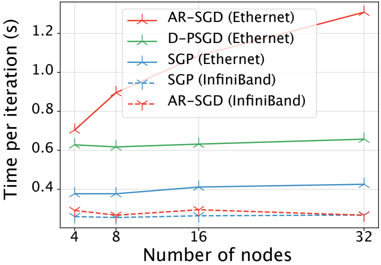
(a) Iteration-wise convergence (b) Time-wise convergence

Figure: Comparison of AllReduce-SGD (AR-SGD), SGP and D-PSGD on 8–16 nodes interconnected via 10 Gbps Ethernet. All methods are run for 90 epochs.

Remarks:

- Increasing $\sharp$ nodes by 2 $\Rightarrow$ $\sharp$ iterations is decreasing by 2.
- SGP completes 90 epochs in less time than other methods.

Scaling and Convergence



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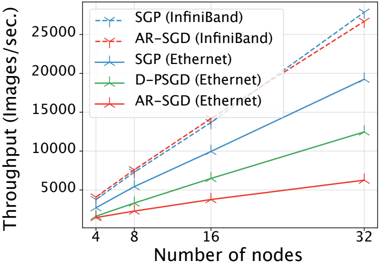

(b)

Figure: Comparison of AllReduce-SGD (AR-SGD), SGP and D-PSGD on 4–32 nodes interconnected via 10 Gbps Ethernet and 100Gbps-InfiniBand.

Remarks:

- InfiniBand: all methods, constant time per iteration
- Ethernet: SGP is the faster method (1.5x faster than D-PSGD)

Scaling and Convergence

	4 nodes (32 GPUs)		8 nodes (64 GPUs)		16 nodes (128 GPUs)		32 nodes (256 GPUs)	
AR-SGD	76.2%	22.0 hrs.	76.4%	14.0 hrs.	76.3%	8.5 hrs.	76.2%	5.1 hrs.
D-PSGD	76.4%	19.7 hrs.	76.1%	9.7 hrs.	75.9%	5.0 hrs.	74.4%	2.6 hrs.
SGP	76.3%	11.8 hrs.	76.4%	5.9 hrs.	75.9%	3.2 hrs.	75.0%	1.7 hrs.

Table: Top-1 validation accuracy (%) and training time (hours), when communicating over 10 Gbps Ethernet for AR-SGD, SGP and D-PSGD. SGP is using 1-peer communication topology. All methods are run for 90 epochs.

- SGP outperforms D-PSGD and AllReduce-SGD in terms of total training time
- Validation accuracy degrades for larger networks (16-32 nodes)

Communication and the speed-accuracy tradeoff.

We explore the effect of communication topology on the speed-accuracy tradeoff (16-31 nodes).

	16 nodes (128 GPUs)		**32 nodes (256 GPUs)**	
AR-SGD	76.3%	8.5 hrs.	76.2%	5.2 hrs.
2P-SGP	76.2%	5.1 hrs.	75.7%	2.5 hrs.
1P-SGP	75.9%	3.2 hrs.	75.0%	1.7 hrs.
AR/1P-SGP	76.2%	4.8 hrs.	75.4%	2.8 hrs.
2P/1P-SGP	76.0%	3.5 hrs.	75.1%	1.8 hrs.

Table: Top-1 validation accuracies (%) and training time (hours) for 1P-SGP (1-peer topology); 2P-SGP (2-peer topology), AR-SGD (AllReduce SGD), AR/1P-SGP (AllReduce first 30 epochs, 1-peer topology last 60 epochs), and 2P/1P-SGP (2-peer topology first 30 epochs, 1-peer topology last 60 epochs), all communicating over 10 Gbps Ethernet.

Overlap SGP (O-SGP)

	Train Acc.	Val. Acc.	Train Time
AR-SGD	76.9%	76.3%	8.5 hrs.
D-PSGD	75.6%	75.9%	4.9 hrs.
AD-PSGD	74.7%	75.5%	2.9 hrs.
SGP	75.6%	75.9%	3.2 hrs.
1-OSGP	77.1%	75.7%	1.8 hrs.

Table: Comparing state-of-the-art synchronous and asynchronous gossip-based approaches to 1-OSGP, an implementation of synchronous SGP where communication is overlapped with 1 gradient step (all messages are always received with 1-iteration of staleness). Experiments are run for 90 epochs over 16 nodes (128 GPUs) interconnected via 10 Gbps Ethernet.

- Overlapping communication and computation: 1)*speeds up training* and 2)*no accuracy degradation*.
- synchronous 1-OSGP is faster than asynchronous AD-PSGD, and achieves better training and validation accuracy.

Fixed runtime budget.

We now compare the methods based on *runtime budget* and not epoch budget.

	Train Acc.	Val. Acc.	Train Time
AR-SGD	76.9%	76.2%	5.1 hrs. (90 epochs)
AD-PSGD	80.3%	76.9%	4.7 hrs. (270 epochs)
SGP	80.0%	77.1%	4.6 hrs. (270 epochs)
1-OSGP	81.8%	77.1%	2.7 hrs. (270 epochs)

Table: Comparing AllReduce SGD (AR-SGD) and SGP under a fix runtime budget. Experiments are run over 1-peer graph topologies, using 32 nodes (256 GPUs) interconnected via 10 Gbps Ethernet.

- Given a similar runtime, SGP outperforms AR-SGD for both training and validation accuracy.
- Running 1-OSGP for the same number of epochs than SGP outperforms SGD while improving the overal training efficiency.

Conclusion

Conclusion

- SGP and O-SGP for accelerating distributed training of DNNs.
- Theoretical convergence guarantees in the smooth non-convex setting, matching known convergence rates for parallel SGD.
- Extensive Numerical Experiments.

Future Directions

Combining techniques for accelerating distributed training of DNNs:

- Compressed messages (Alistarh et al.,2017; Wen et al.,2017; Jia et al.,2018; Koloskova et al.,2019, Tang et al. 2019)
- Truly asynchronous gossip-based variant (Jin et al.,2016 ; Lian et al., 2018)

Extensions on analysis:
Provide analysis for the momentum variant, stage-wise learning rate, remove strong assumptions.

Thank You!